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2000 J. Phys. A: Math. Gen. 33 5011

(http://iopscience.iop.org/0305-4470/33/28/305)

J. Phys. A: Math. Gen. **33** (2000) 5011–5016. Printed in the UK PII: S0305-4470(00)09991-1

Generalized Maxwell equations from the Einstein postulate

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Received 29 November 1999, in final form 25 May 2000

Abstract. We show that the Gersten derivation of Maxwell equations can be generalized. It actually leads to additional solutions of '$S = 1$ equations'. They follow directly from previous considerations by Majorana, Oppenheimer (Oppenheimer J R 1931 *Phys. Rev.* **38** 725), Weinberg (Weinberg S 1964 *Phys. Rev.* **134** B882) and Ogievetskiĭ and Polubarinov (Ogievetskiĭ V I and Polubarinov I V 1966 *Yadern. Fiz.* **4** 216 (Engl. Transl. 1967 *Sov. J. Nucl. Phys.* **4** 156)). Therefore, *generalized* Maxwell equations should be used as a guideline for proper interpretations of quantum theories.

In his paper [1] Gersten studied the matrix representation of the Maxwell equations, both the Faraday and Ampére laws and the Gauss law. His consideration is based on equation (9) of [1]:

$$\left(\frac{E^2}{c^2} - \boldsymbol{p}^2\right)\boldsymbol{\Psi} = \left(\frac{E}{c}I^{(3)} - \boldsymbol{p}\cdot\boldsymbol{S}\right)\left(\frac{E}{c}I^{(3)} + \boldsymbol{p}\cdot\boldsymbol{S}\right)\boldsymbol{\Psi} - \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}(\boldsymbol{p}\cdot\boldsymbol{\Psi}) = 0. \qquad (1)$$

Furthermore, he claimed that the solutions to this equation should be found from the set

$$\left(\frac{E}{c}I^{(3)} + \boldsymbol{p}\cdot\boldsymbol{S}\right)\boldsymbol{\Psi} = 0 \qquad \text{equation (10) of [1]}$$

$$(\boldsymbol{p}\cdot\boldsymbol{\Psi}) = 0 \qquad \text{equation (11) of [1].}$$

Thus, Gersten concluded that his equation (9) is equivalent to his Maxwell equations (10) and (11). As he also correctly indicated, such a formalism for describing $S = 1$ fields has been considered by several authors before. See, for instance, [2–10]; those authors mainly considered the dynamical Maxwell equations in the matrix form.

However, we straightforwardly note that equation (9) of [1] is also satisfied under the choice†

$$\left(\frac{E}{c}I^{(3)} + \boldsymbol{p}\cdot\boldsymbol{S}\right)\boldsymbol{\Psi} = \boldsymbol{p}\chi \qquad (2)$$

$$(\boldsymbol{p}\cdot\boldsymbol{\Psi}) = \frac{E}{c}\chi \qquad (3)$$

with some arbitrary scalar function χ at this stage. This is due to the fact that‡

$$(\boldsymbol{p}\cdot\boldsymbol{S})^{jk}\boldsymbol{p}^k = \mathrm{i}\epsilon^{jik}p^i p^k \equiv 0$$

† We leave the analysis of possible functional nonlinear (in general) dependence of χ and $\partial_\mu\chi$ on the higher-rank tensor fields for future publications.

‡ See the explicit form of the angular momentum matrices in equation (6) of Gersten's paper [1].

(or after substitutions of quantum operators rot grad $\chi = 0$). Thus, the generalized coordinate-space Maxwell equations follow, after a similar procedure as in [1]:

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \mathrm{Im}\,\chi \tag{4}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \nabla \mathrm{Re}\,\chi \tag{5}$$

$$\nabla \cdot \boldsymbol{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathrm{Re}\,\chi \tag{6}$$

$$\nabla \cdot \boldsymbol{B} = \frac{1}{c}\frac{\partial}{\partial t}\mathrm{Im}\,\chi. \tag{7}$$

If one assumes that there are no monopoles, one may suggest that $\chi(x)$ is a *real* field and its derivatives play the role of charge and current densities. Thus, surprisingly, on using the Dirac-like procedure† of derivation of 'free-space' relativistic quantum field equations, Gersten might in fact have come to the *inhomogeneous* Maxwell equations‡! Furthermore, I am not aware of any proofs that the scalar field $\chi(x)$ must be firmly connected with the charge and current densities, so there is sufficient room for interpretation. For instance, its time derivative and gradient may also be interpreted as leading to the 4-vector potential. In this case, we need some *mass/length* parameter as in Lyttleton and Bondi [11] and Chambers [11]. Both these interpretations were present in the literature [9, 11] (cf also [12]).

Furthermore, Gersten's equation (9), which is our equation (1), is satisfied also when

$$\left(\frac{E}{c}I^{(3)} - \boldsymbol{p}\cdot\boldsymbol{S}\right)\tilde{\boldsymbol{\Psi}} = \boldsymbol{p}\tilde{\chi} \tag{8}$$

$$(\boldsymbol{p}\cdot\tilde{\boldsymbol{\Psi}}) = \frac{E}{c}\tilde{\chi} \tag{9}$$

for some function $\tilde{\boldsymbol{\Psi}} = \boldsymbol{C} + \mathrm{i}\boldsymbol{D}$. The corresponding Maxwell-like equations are

$$\nabla \times \boldsymbol{C} = -\frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} - \nabla \mathrm{Im}\,\tilde{\chi} \tag{10}$$

$$\nabla \times \boldsymbol{D} = \frac{1}{c}\frac{\partial \boldsymbol{C}}{\partial t} + \nabla \mathrm{Re}\,\tilde{\chi} \tag{11}$$

$$\nabla \cdot \boldsymbol{C} = -\frac{1}{c}\frac{\partial}{\partial t}\mathrm{Re}\,\tilde{\chi} \tag{12}$$

$$\nabla \cdot \boldsymbol{D} = -\frac{1}{c}\frac{\partial}{\partial t}\mathrm{Im}\,\tilde{\chi}. \tag{13}$$

Unless $\tilde{\chi} = \chi^*$ we cannot make the identification $\tilde{\boldsymbol{\Psi}} = \boldsymbol{\Psi}^* = \boldsymbol{E} + \mathrm{i}\boldsymbol{B}$. Thus the general solution of the '$S = 1$ equation' is the superposition $\boldsymbol{\Psi}^{\mathrm{tot}} = c_1\boldsymbol{\Psi} + c_2\tilde{\boldsymbol{\Psi}}$, with $\boldsymbol{\Psi}$ and $\tilde{\boldsymbol{\Psi}}$ being the subject of constraints (4)–(7), (10)–(13). Moreover, from a physical point of view this

† That is to say, on the basis of the relativistic dispersion relations

$$(E^2 - c^2\boldsymbol{p}^2 - m^2c^4)\Psi = 0 \qquad \text{equation (1) of [1].}$$

‡ One can also substitute $-(4\pi\mathrm{i}\hbar/c)\boldsymbol{j}$ and $(-4\pi\mathrm{i}\hbar)\rho$ in the right-hand side of (2) and (3) of this paper and obtain equations for the current and the charge density

$$\frac{1}{c}\nabla \times \boldsymbol{j} = 0$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{j}}{\partial t} + \nabla\rho = 0$$

which coincide with equations (13) and (17) of Dvoeglazov [9]. The interesting question is whether such defined $\boldsymbol{j}$ and ρ may be related to $\partial_\mu\chi$.

may signify that the present-day experimental unobservability of magnetic monopoles can have a deeper theoretical reason in the mutual cancellation of the corresponding terms in the divergence equation for the magnetic part of the total '$S = 1$' field by means of appropriately fixing the chi-functions.

Below we discuss only one aspect of the above-mentioned problem with additional scalar field and its derivatives in generalizations of the Maxwell formalism. It is connected with the concept of *notoph* of Ogievetskiĭ and Polubarinov (in the US journal literature this is known as the Kalb–Ramond field)†. The related problem of misunderstandings of the Weinberg theorem $B - A = \lambda$ is briefly discussed too; A and B are eigenvalues of angular momenta corresponding to a certain representation of the Lorentz group and λ is the helicity [4, p B885].

Actually, after performing the Bargmann–Wigner procedure of description of higher-spin massive particles by a totally symmetric spinor of higher rank, we derive the following equations for spin 1:

$$\partial_\alpha F^{\alpha\mu} + \frac{m}{2} A^\mu = 0 \tag{14}$$

$$2mF^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{15}$$

In the meantime, in the textbooks, the latter set is usually written as

$$\partial_\alpha F^{\alpha\mu} + m^2 A^\mu = 0 \tag{16}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{17}$$

The set (16), (17) is obtained from (14), (15) after the normalization change $A_\mu \to 2mA_\mu$ or $F_{\mu\nu} \to \frac{1}{2m} F_{\mu\nu}$. Of course, one can investigate other sets of equations with different normalization of the $F_{\mu\nu}$ and A_μ fields. 'Are all these sets of equations equivalent?' I asked in a recent series of papers.

Ogievetskiĭ and Polubarinov [13] argued that in the massless limit 'the system of $2s + 1$ states is no longer irreducible; it decomposes and describes a set of different particles with zero mass and helicities $\pm s, \pm(s-1), \ldots, \pm 1, 0$ (for integer spin and if parity is conserved; the situation is analogous for half-integer spins)'. Thus, they did in fact contradict the Weinberg theorem. But, in [15] I presented explicit forms of 4-vector potentials and of parts of the antisymmetric tensor (AST) field and concluded that the question should be solved on the basis of the analysis of normalization issues. Here they are in the momentum representation:

$$u^\mu(\boldsymbol{p}, +1) = -\frac{N}{\sqrt{2}m} \begin{pmatrix} p_r \\ m + \frac{p_1 p_r}{E_p+m} \\ \mathrm{i}m + \frac{p_2 p_r}{E_p+m} \\ \frac{p_3 p_r}{E_p+m} \end{pmatrix} \qquad u^\mu(\boldsymbol{p}, -1) = \frac{N}{\sqrt{2}m} \begin{pmatrix} p_l \\ m + \frac{p_1 p_l}{E_p+m} \\ -\mathrm{i}m + \frac{p_2 p_l}{E_p+m} \\ \frac{p_3 p_l}{E_p+m} \end{pmatrix} \tag{18}$$

$$u^\mu(\boldsymbol{p}, 0) = \frac{N}{m} \begin{pmatrix} p_3 \\ \frac{p_1 p_3}{E_p+m} \\ \frac{p_2 p_3}{E_p+m} \\ m + \frac{p_3^2}{E_p+m} \end{pmatrix} \qquad u^\mu(\boldsymbol{p}, 0_t) = \frac{N}{m} \begin{pmatrix} E_p \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} \tag{19}$$

and

$$\boldsymbol{B}^{(+)}(\boldsymbol{p}, +1) = -\frac{\mathrm{i}N}{2\sqrt{2}m} \begin{pmatrix} -\mathrm{i}p_3 \\ p_3 \\ \mathrm{i}p_r \end{pmatrix} = +\mathrm{e}^{-\mathrm{i}\alpha_{-1}} \boldsymbol{B}^{(-)}(\boldsymbol{p}, -1) \tag{20}$$

† In my opinion, Weinberg [14, p 208] partly confirmed this idea when considering a spin-0 4-vector field in his well known book.

$$\boldsymbol{B}^{(+)}(\boldsymbol{p},0)=\frac{\mathrm{i}N}{2m}\begin{pmatrix}p_2\\-p_1\\0\end{pmatrix}=-\mathrm{e}^{-\mathrm{i}\alpha_0}\boldsymbol{B}^{(-)}(\boldsymbol{p},0) \quad (21)$$

$$\boldsymbol{B}^{(+)}(\boldsymbol{p},-1)=\frac{\mathrm{i}N}{2\sqrt{2}m}\begin{pmatrix}\mathrm{i}p_3\\p_3\\-\mathrm{i}p_l\end{pmatrix}=+\mathrm{e}^{-\mathrm{i}\alpha_{+1}}\boldsymbol{B}^{(-)}(\boldsymbol{p},+1) \quad (22)$$

and

$$\boldsymbol{E}^{(+)}(\boldsymbol{p},+1)=-\frac{\mathrm{i}N}{2\sqrt{2}m}\begin{pmatrix}E_p-\frac{p_1p_r}{E_p+m}\\\mathrm{i}E_p-\frac{p_2p_r}{E_p+m}\\-\frac{p_3p_r}{E+m}\end{pmatrix}=+\mathrm{e}^{-\mathrm{i}\alpha'_{-1}}\boldsymbol{E}^{(-)}(\boldsymbol{p},-1) \quad (23)$$

$$\boldsymbol{E}^{(+)}(\boldsymbol{p},0)=\frac{\mathrm{i}N}{2m}\begin{pmatrix}-\frac{p_1p_3}{E_p+m}\\-\frac{p_2p_3}{E_p+m}\\E_p-\frac{p_3^2}{E_p+m}\end{pmatrix}=-\mathrm{e}^{-\mathrm{i}\alpha'_0}\boldsymbol{E}^{(-)}(\boldsymbol{p},0) \quad (24)$$

$$\boldsymbol{E}^{(+)}(\boldsymbol{p},-1)=\frac{\mathrm{i}N}{2\sqrt{2}m}\begin{pmatrix}E_p-\frac{p_1p_l}{E_p+m}\\-\mathrm{i}E_p-\frac{p_2p_l}{E_p+m}\\-\frac{p_3p_l}{E_p+m}\end{pmatrix}=+\mathrm{e}^{-\mathrm{i}\alpha'_{+1}}\boldsymbol{E}^{(-)}(\boldsymbol{p},+1) \quad (25)$$

where we denoted a normalization factor appearing in the definitions of the potentials (and/or in the definitions of the physical fields through potentials) as N (which can, of course, be chosen in an arbitrary way, not necessarily to be proportional to m)† and $p_{r,l}=p_1\pm\mathrm{i}p_2$. Thus, we find that in the massless limit we may have in general divergent parts of 4-potentials and the AST field, thus prohibiting setting $m=0$ in the equations (14)–(17). They are usually removed by 'electrodynamic' gauge transformations, but it was shown that the Lagrangian constructed from the $(1,0)\oplus(0,1)$ (or AST) fields admits another kind of 'gauge' transformation, namely $(F_{\mu\nu}\to F_{\mu\nu}+\partial_\nu\Lambda_\mu-\partial_\mu\Lambda_\nu)$, with some 'gauge' vector functions Λ_μ. This becomes the origin of the possibility of obtaining the quantum states (particles?) of different helicities in both the $(\frac{1}{2},\frac{1}{2})$ and $(1,0)\oplus(0,1)$ representations.

In our formulation of generalized Maxwell equations these in general divergent terms can be taken into account explicitly, thus giving additional terms in (4)–(7). As suggested in [11] they may be applied to explanations of several cosmological puzzles. The detailed analysis of contradictions between the Weinberg theorem and the Ogievetskiĭ–Polubarinov–Kalb–Ramond conclusion (and also discussions of [16]) will be given in a separate publication. Here, I would only like to mention a few assumptions, under which Weinberg derived his famous theorem.

- The derivation is based on the analysis of the proper Lorentz transformations only. The discrete symmetry operations of the full Poincaré group (which, for instance, may lead to the change of the sign of the energy) have not been considered there. Nor have normalization transformations been considered. However, S Weinberg noted (see the fourth line from the bottom in [4, p B885]) that '(the photon cannot) be associated with vector potential $(\frac{1}{2},\frac{1}{2}),\ldots$ until we broaden our notion what we mean by a Lorentz transformation'.
- Weinberg used the non-semi-simple structure of the little group for $m=0$ particles.
- The derivation assumes a particular choice of the coordinate frame, namely $\boldsymbol{p}=(0,0,p_3)$ and $p_3=|\boldsymbol{p}|$‡.

† The possibility of appearance of additional mass factors in commutation relations was also analysed by us in the recent series of papers.

‡ As one can see, unpolarized classical $\boldsymbol{E}$ and $\boldsymbol{B}$ depend indeed on the choice of the coordinate system, equations (21) and (24).

- The derivation does *not* assume that the AST field is related to 4-vector fields by a certain derivative operator. Neither does it present its explicit forms (obtained from the Bargmann–Wigner procedure, for instance) or normalization fixing of this 4-vector.

Finally, the intrinsic angular momentum operator of the electromagnetic field (which can be found on the basis of the Noether theorem) contains the coefficient functions which belong to different representations of the Lorentz group, $\boldsymbol{S} \sim \boldsymbol{E} \times \boldsymbol{A}$, and it acts in the Fock space [15]. Furthermore, the condition (35) $W^{\mu} = kp^{\mu}$ is *not* the only condition which can be imposed for massless particles. Namely, as stressed by Korff in [16] the Pauli–Lubanski vector *may* be a spacelike vector in this case, which would correspond to an 'infinite-spin' representation.

Finally, we would like to add some words to the Dirac derivation of equations (30)–(33) of the Gersten paper [1] and their analysis. We derived the formula (for spin 1)

$$[\boldsymbol{S}^i(\boldsymbol{S}\cdot\boldsymbol{p})]^{jm} = [\boldsymbol{p}^i I^{jm} - \mathrm{i}[\boldsymbol{S}\times\boldsymbol{p}]^{i,jm} - p^m\delta^{ij}] \tag{26}$$

with i being the vector index and j, m being the matrix indices. Hence, from the equation ($k = 1$)

$$\{kp_t + S_x p_x + S_y p_y + S_z p_z\}\psi = 0 \tag{27}$$

multiplying subsequently by S_x, S_y and S_z one can obtain in the case $S = 1$

$$\{p_x + S_x p_t - \mathrm{i}S_y p_z + \mathrm{i}S_z p_y\}^{jm}\psi^m - (\boldsymbol{p}\cdot\boldsymbol{\psi})\delta^{xj} = 0 \tag{28}$$

$$\{p_y + S_y p_t - \mathrm{i}S_z p_x + \mathrm{i}S_x p_z\}^{jm}\psi^m - (\boldsymbol{p}\cdot\boldsymbol{\psi})\delta^{yj} = 0 \tag{29}$$

$$\{p_z + S_z p_t - \mathrm{i}S_x p_y + \mathrm{i}S_y p_x\}^{jm}\psi^m - (\boldsymbol{p}\cdot\boldsymbol{\psi})\delta^{zj} = 0. \tag{30}$$

One can see from the above that the equations (31)–(33) of [1] can be considered as the consequence of equation (30) of [1] and additional the 'transversality condition' $\boldsymbol{p}\cdot\boldsymbol{\psi} = 0$ in the case of the spin-1 consideration. So, it is not surprising that they are equivalent to the complete set of Maxwell's equations. They are obtained after multiplications by corresponding $\boldsymbol{S}$ matrices. However, the crucial mathematical problem with such a multiplication is that the $\boldsymbol{S}$ matrices for boson spins are *singular*, $\det S_x = \det S_y = \det S_z \equiv 0$, which makes the above procedure doubtful† and leaves room for possible generalizations. Moreover, the right-hand side of equation (30) of [1] may also be different from zero according to our analysis above.

The conclusion of my paper is that, unfortunately, the possible consequences following from Gersten's equation (9) have not been explored in full; on this basis we would like to correct his conclusion and his claim in the abstract of [1]. It is the *generalized* Maxwell equations (many versions of which have been proposed during the last 100 years, see, for instance, [17]) that should be used as a guideline for proper interpretations of quantum theories.

Acknowledgments

I greatly appreciate discussions with Professor A Chubykalo, Professor L Horwitz and Professor A Gersten, and the useful information from Professor D Ahluwalia (even though I disagree with his methods in science), Professor A F Pashkov, Professor E Recami, Professor M Sachs and an anonymous referee of *J. Phys. A: Math. Gen.* Zacatecas University, Mexico, is thanked for awarding the professorship. This work has been partly supported by the Mexican Sistema Nacional de Investigadores and the Programa de Apoyo a la Carrera Docente.

† After an analysis of the literature, I learnt that, unfortunately, a similar procedure has been applied in the derivation of many higher-spin equations without proper explanation and precaution.

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